Network Analysis (PERT / CPM)

Network analysis refers to a no. of techniques for the planning and control complex projects. The basis of network planning is the presentation of sequential relationships between activities by means of lines and circles. The idea is to link the various activities in such a way that the overall time spent on the project is kept to a minimum. The optimum linking of various stages is called the critical path.

Advantages :- (1) They provide a logical picture of the layout and sequence of a complex project.

(2) They help to identify the events which are critical to entire project.

(3) They help us to find out cost, time, resources etc involved in the project.

(4) They help us in coordination of various activities.

(5) They help in planning and control of complex projects.

The two important techniques, which are frequently used in planning are PERT (Programme evaluation and review technique) and CPM (critical path method)

PERT :- It is one of the techniques used in projects which are of non-repetitive in nature, and for those projects in which precise time determination can not be made

(P.T.O)

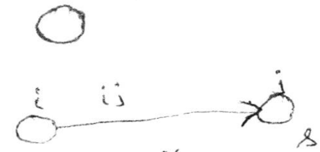
In such projects, mgt. can not be guided by the past experience. For ex:- The project of launching a space craft involves the work which is not done before. For such projects the range of technical problem is immense and time estimates are mere guess works. So PERT is best suited for those projects.

CPM:- These are used for repetitive projects for which ~~and~~ accurate estimates can be made for each activity and cost estimates also can be made with fair degree of accuracy. But it is not suitable for R&D

PERT

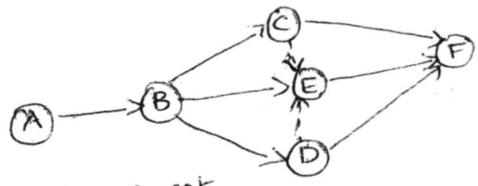
Activity.
Tail → Head
task begins — task ends

Event indicates the start / end of an activity



ij – activity connecting two events.

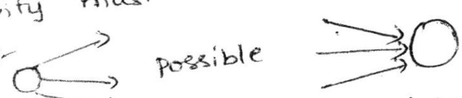
Network: combⁿ of activities, dummy activities, and events in a logical sequence, according to the rules for drawing networks



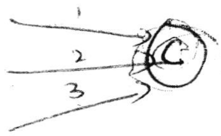
Rules

1. only one start and one end pt
2. activity must have one tail event and one head event.

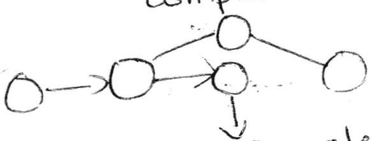
Divergent — possible; possible (convergent)



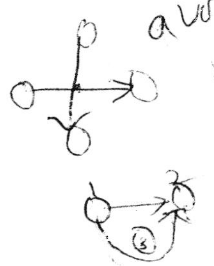
3. until tail event is completed, activity cannot start.
4. Event C is not completed until activities 1, 2, 3 are completed.



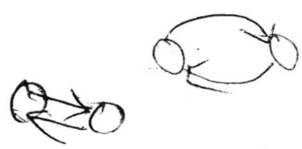
5. Dangler (avoid them) (untied events)



avoid



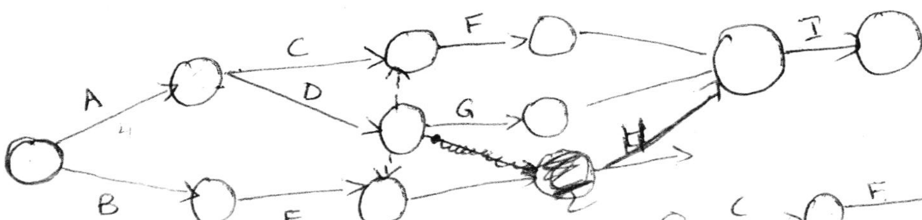
6. Better to avoid intersection
7. no curved arrows, always from left to right
8. length should be uniform – no effect on time duration
9. Loop should be avoided

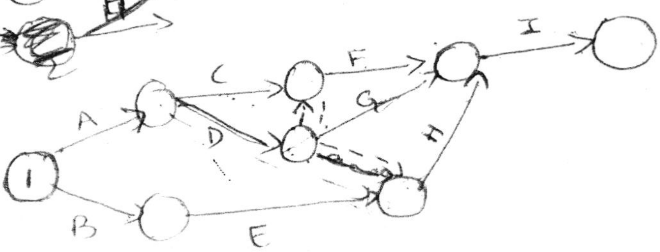


10. Arrow cannot go backward
11. use dummies only when needed (clear explanation of sequence)

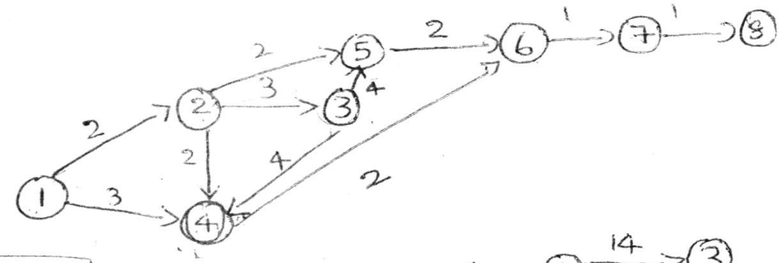
simple networks – events are numbered 1, 2, 3 . . .
large " – 10, 20, 30, . . . additional activities can be inserted easily

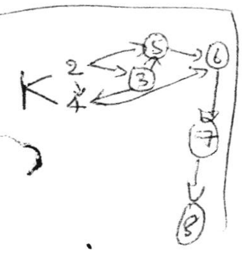
18.1



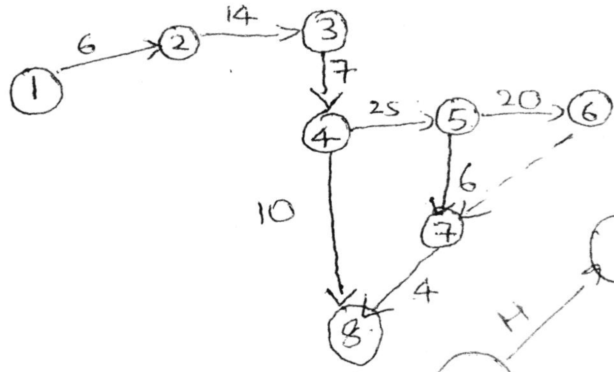


18.2





18.3



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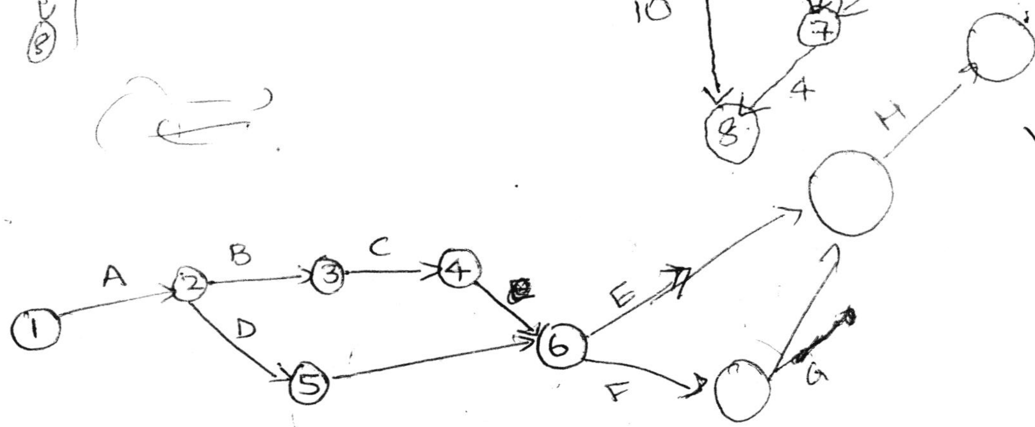
18.4



Avg time estimate $(t_e) = \frac{1}{6} t_o + \frac{2}{3} t_m + \frac{1}{6} t_p$

18.5

$t_e = \frac{1}{6}(20) + \frac{2}{3}(30) + \frac{1}{6}(40)$

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

$= \frac{20+120+40}{6} = \frac{180}{6} = 30$ mins

Range $= (t_p - t_o)$

$\sigma = \frac{t_p - t_o}{6}$

$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$ → lesser Variance is reliable.

Critical path — Path which consumes the maximum amount of time or resources.

Path with zero slack.

slack $= T_L - T_E$

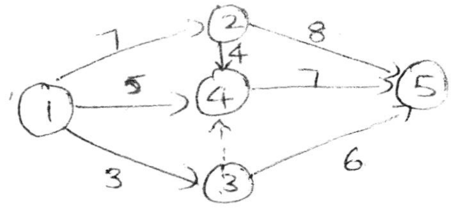
(T_L: Latest allowable occurance time) (T_E: Earliest Expected time)

forward pass ↘ T_E = max value of (T_E(Predecessor) + t_e(activity)

backward pass ↙ T_L (Predecessor) = min of (T_L(Successor) − t_e }

Slack mean the time taken to delay a particular event comfortably without affecting the project completion time. If it has zero slack then it is critical path

18.7

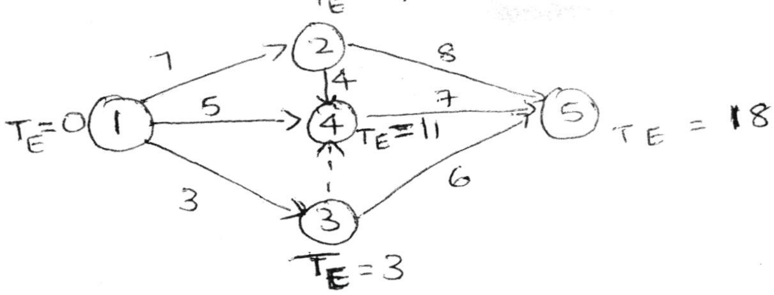


18.8

$T_L = 20$

$T_E = 18$

Slack = 20 − 18 = 2 days



Float It is better to have some idle resources as a safety measure. represents underutilised ~~activities~~ resources

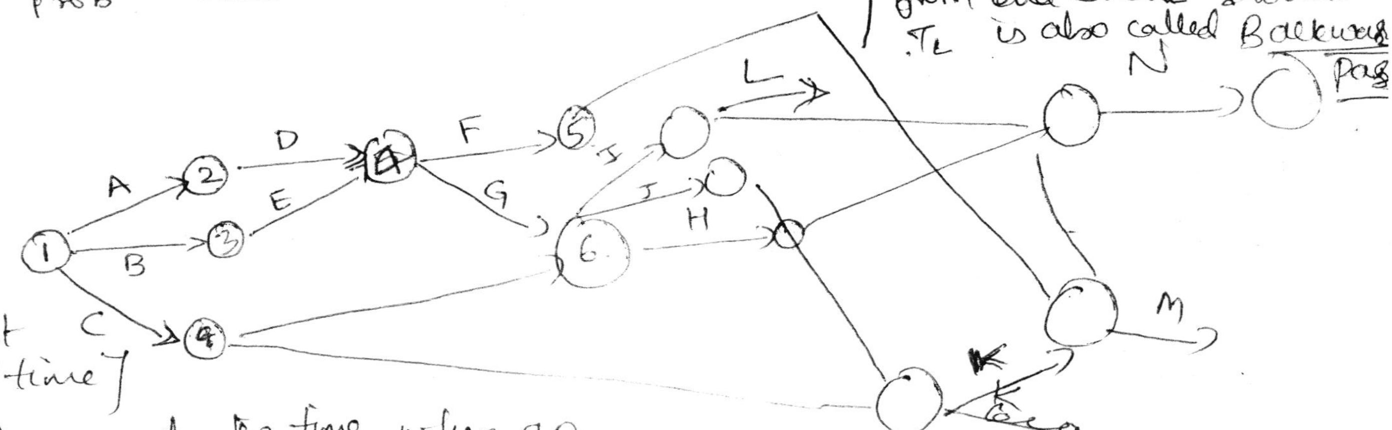
A critical path in a network is that path which has least float.

Prob of completing the project within the given time:

1. Find range $(t_p - t_o)$
2. Determine σ for each activity
3. " σ^2
4. Find $\Sigma\sigma^2 = \sigma_1^2 + \sigma_2^2 + \dots + \sigma_n^n$
5. Determine $\sigma = \sqrt{\Sigma\sigma_1^2 + \sigma_2^2 + \dots + \sigma_n^n}$
6. Divide slack by σ $\quad Z = \frac{T_S - T_E}{\sigma} \quad \pm 3\sigma$
7. prob value.

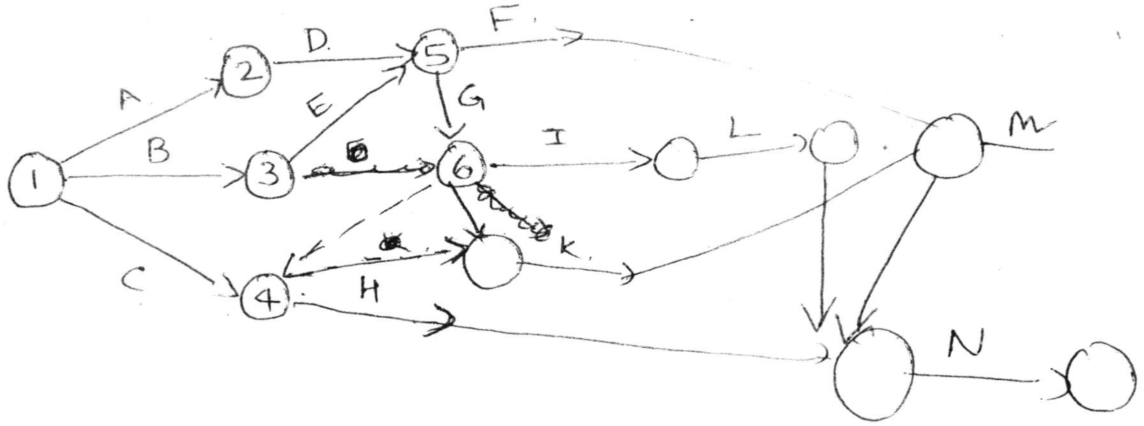
Latest allowable Occurrence time (T_L)

It is the latest time by which an event must ~~be~~ occur to keep the project on schedule. If not project gets delayed. Computation (T_L) we start from end event & workout. T_L is also called Backward Pass



[Earliest Expected time]

(T_E) ~~refers to~~ the time when an event can be expected to be completed at the earliest. It is computed by adding of te's of activity path leading to that Event. starts with start event & worked out for all events so it is called as forward pass



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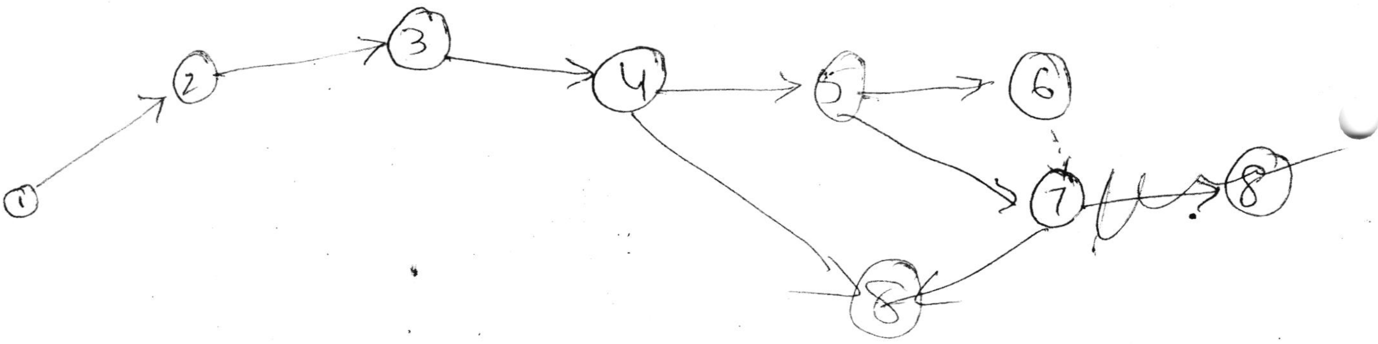

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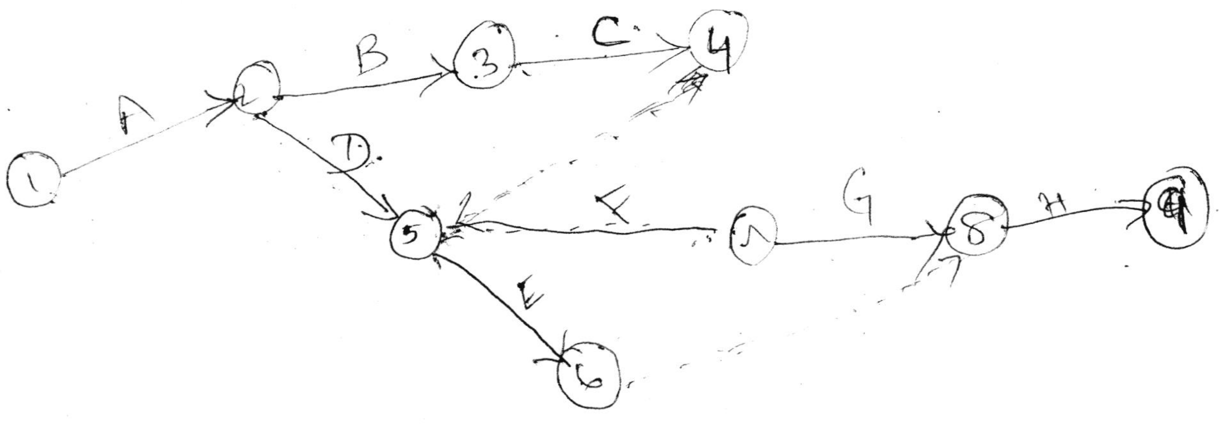

A
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INTRO

Purpose of PERT analysis — is to ascertain the probability of completing the project within a given time period.

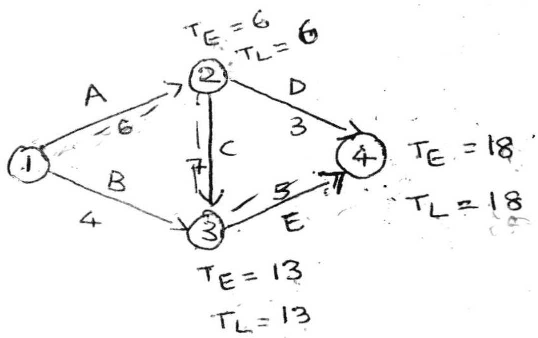
If prob of completing the project within a given time is low or there is urgency to complete the project, it is necessary to find an alternative. The mgmt has only one alternative — to crash the duration of the project by pumping add^nl i/p's of labour and machines. Mgmt would like to optimise the costs and the time.

T_E = 2 hrs } 1 hr
T_L = (3) hrs

Project Costs and Time

- Direct Costs — directly proportional to the number of activities involved
- Indirect Costs — determined per day. Personnel Salaries, rent, supplies, adv . . .

Crash cost — cost incurred to reduce activity duration to its minimum.



Critical path — 1→2→3→4

Normal cost = 5000 + 3000 + 6500 + 4000 + 8500 = Rs27,000

$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal Time} - \text{Crash Time}}$$

Activity	Preceding Activity	Time Normal	Time Crash	Cost Normal	Crash Cost	Cost slope & priority	Remark
✓ A (1,2)	—	6	4	5000	6200	600 (II)	
B (1-3)	—	4	2	3000	3900	450	(not under CP)
✓ C (2-3)	A	7	6	6500	6800	300 (I)	
D (2-4)	A	3	2	4000	4500	500	(not under CP)
✓ E (3-4)	C, B	5	3	8500	10000	750 (III)	
				27000			

Project Crashing

Starting point is critical path.

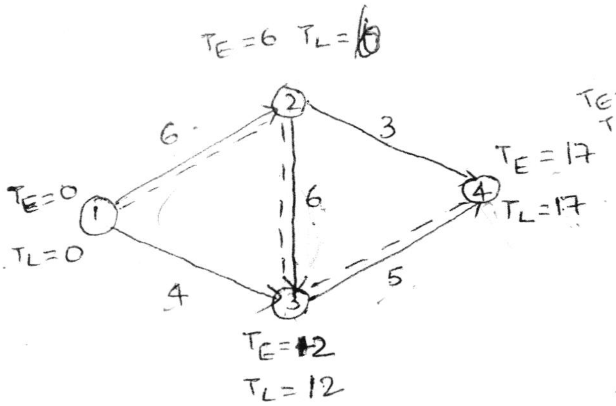
1. CP is identified
2. identify the Priority by Calculating the cost slope
3. network diagram redrawn

$$\text{Cost slope} = \frac{\text{Crash Cost} - \text{N.C}}{\text{N.T} - \text{C.T}}$$

Crash activity 2-3

Stage I

Crash 2-3 by one day

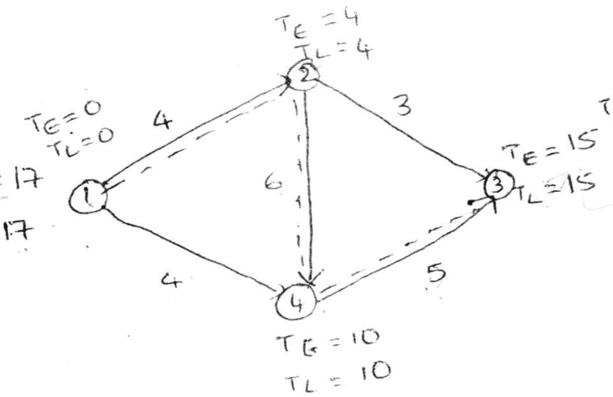


1 - 2 - 3 - 4

N/C = 27000 + 300

Stage II

Crash 1-2 by two days



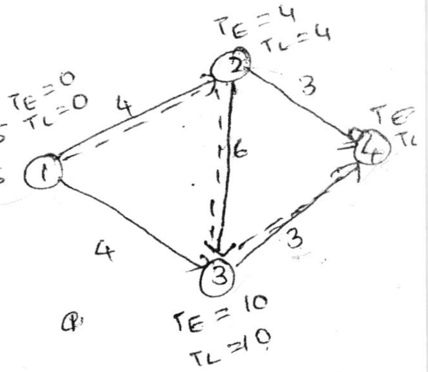
1 - 2 - 3 - 4

N.C = 27300 + 600 (2)

= 28500

Stage III

Crash 3-4 by 2 days



1 - 2 - 3 - 4

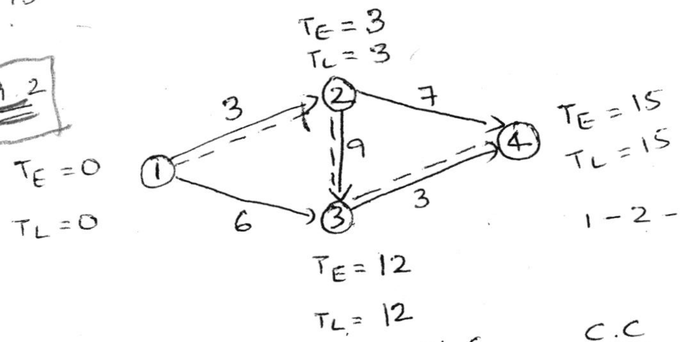
28500 + 750 (2)

= 30000

18 days	— 27,000
17 "	— 27,300
15 "	— 28,500
13 "	— 30,000

Crashing	N.T C.T	N.C	C.S	Total cost
None				27,000
1				27,300
2				28,500
2				30,000

19.2



slack $T_L - T_E = 0$

1 - 2 - 3 - 4 (critical path)

↳ [always longest]

Jobs	N.T	C.T	N.C	C.C	Cost Slope	Priority
1-2	3	2	10000	11000	1000	V
1-3	6	3	6000	8400	800	IV
2-3	9	3	9000	12000	500	II
2-4	7	4 (3 ??)	5000	7800	?? $\frac{2800}{3}$ (933.33)	III
3-4	3	2	3000	3400	400	I
			33000			

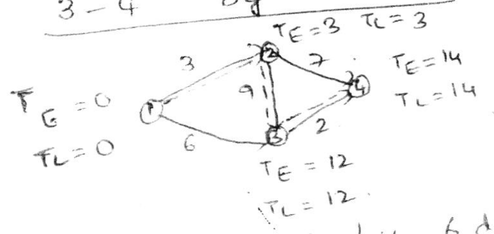
15400
12000
3400

Total cost = 33000 + (15 days × 600)

= Rs 42,000

Crashing 3 - 4 by one day

no change in critical path (1 - 2 - 3 - 4)

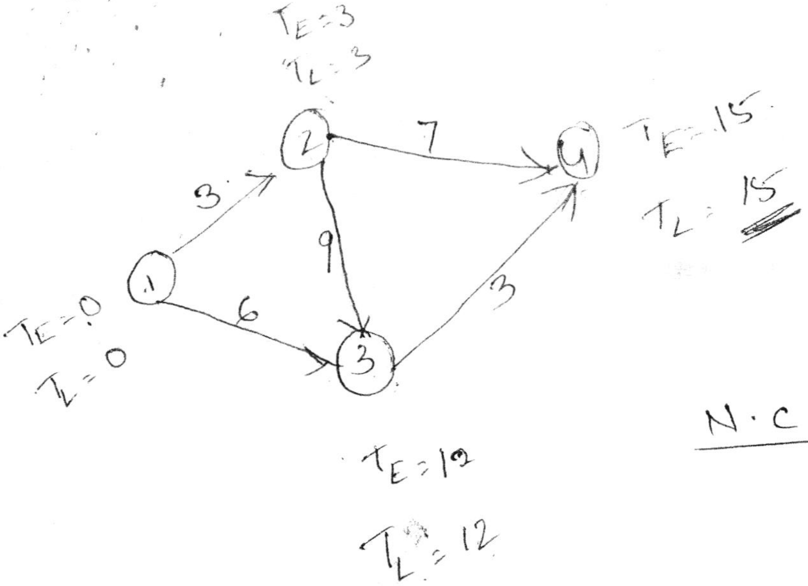


Total cost = (33000 + 400) + (14 days × 600)

= 33400 + 8400

= Rs 41,800

Crashing 2-3 by 6 days



Critical path : 1 - 2 - 3 - 4

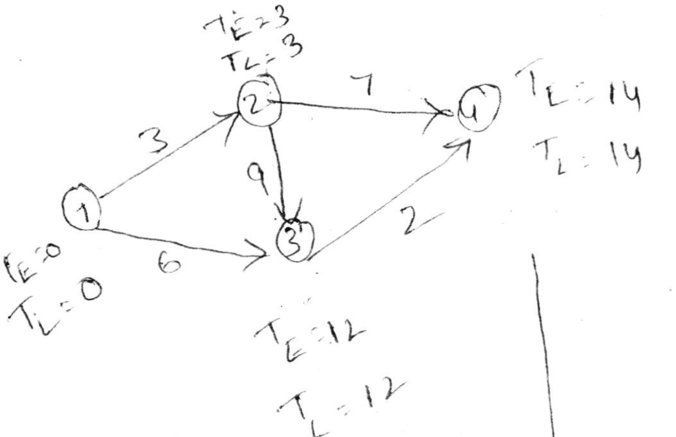
3000/6

$$\text{Cost slope} = \frac{C.C - N.C}{NT - C.T}$$

Jobs	N.T days	Crash Time	N.C	Crash Cost	Cost slope	
1 - 2	3	2	10,000	11,000	1,000	V
1 - 3	6	3	6000	8,400	800	IV
2 - 3	9	3	9000	12,000	500	II
2 - 4	7	3	5000	7,800	700	III
3 - 4	3	2	3000	3,400	400	I
			33,000			

Total cost = 33,000 + (15 x 600) = 42,000

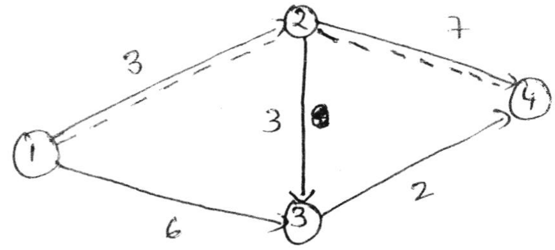
Crash 3-4 by 1 day



1 - 2 - 3 - 4

Total cost = 33,000 (N.C) + 400 (C.slope) + (14 x 600)

= 41,800

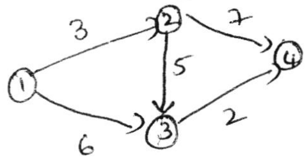


change in critical path (1-2-4)

Total cost = (33000 + 400 + 3000) + (10 × 600)

= 36400 + 6000 = 42400

Crashing 2-3 by 4 days.



1 - 2 - 4
1 - 2 - 3 - 4 } both critical paths

	Jobs	
1 - 2 - 4	1 - 2	Avai
	2 - 4	Avai
1 - 2 - 3 - 4	1 - 2	Avai
	2 - 3	crashed
	3 - 4	crashed

1 - 2 and 2 - 4 ~~affects~~

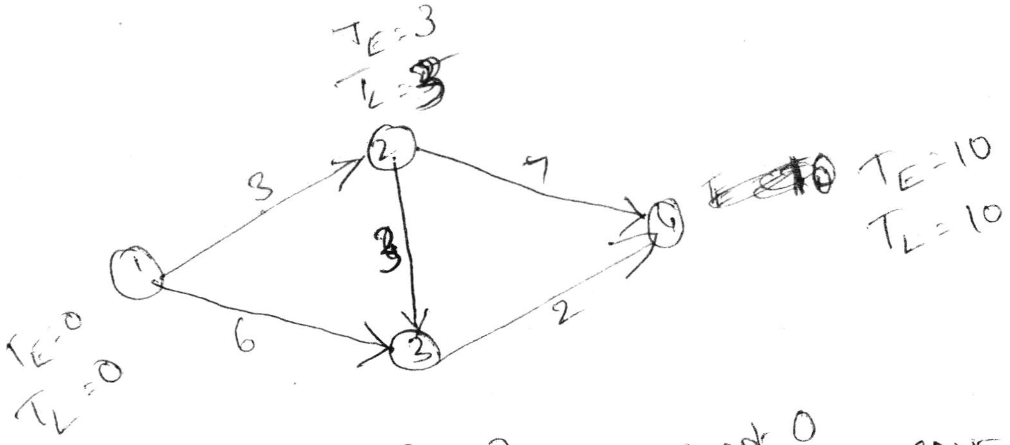
2 - 4 slope is cheaper

2 - 4 won't help 1 - 2 - 3 - 4

So 1 - 2 has to be crashed.

Stage II

Crash 2 - 3 by 6 days.

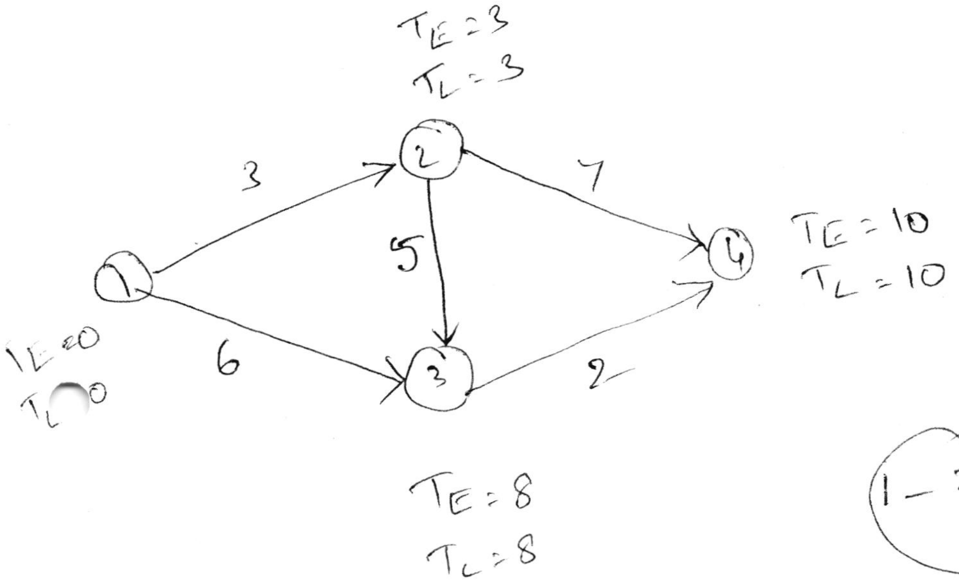


$T_E = 6$, $T_L = 8$ } Slack is not 0 So we will not Consider this event

Critical path

1 - 2 - 4

Some activity must be crashed by 4 days (because 2 days extra expenses we are incurring)



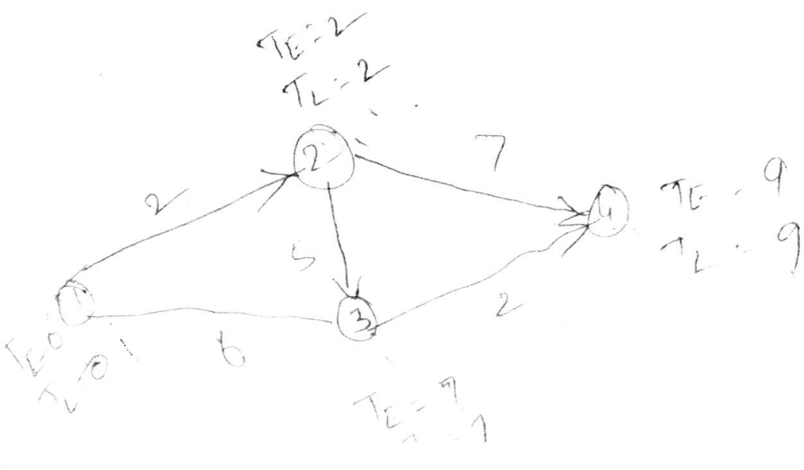
1 - 2 - 4
(or)
1 - 2 - 3 - 4

1 - 2 - 4

Jobs
1 - 2.
2 - 4.

1 - 2 - 3 - 4

1 - 2.
2 - 3
3 - 4



Total Cost =

I. Average time estimate $T_e = \frac{t_0 + 4t_m + t_p}{6}$

t_0 = optimistic, t_m = most likely

t_p = pessimistic time estimates.

$t_0 = 20$, $t_m = 30$, $t_p = 40$; $t_e = \frac{20 + 4(30) + 40}{6}$

$t_e = \frac{20 + 120 + 40}{6} = \frac{180}{6} = 30$

II Range = $(t_p - t_0)$

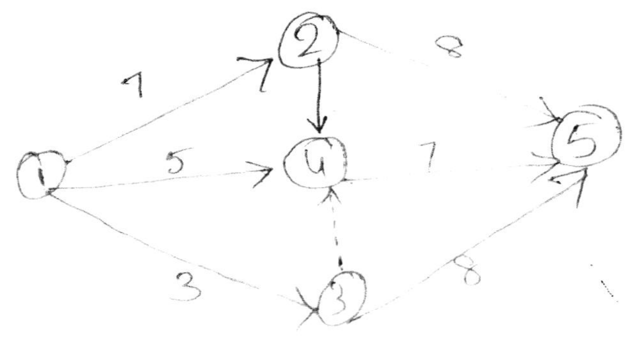
III $\sigma = \frac{(t_p - t_0)}{6}$; $\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$

IV <u>critical path</u> :- It is that path which consumes the maximum amount of time & resources. Critical path will have zero slack. Slack means the time taken to delay a particular event without affecting the project completion time.

slack = $T_L - T_E$; T_L : Latest allowable occurrence time

T_E = the earliest expected time

IV calculate Earliest Expected time (T_E) for given network.



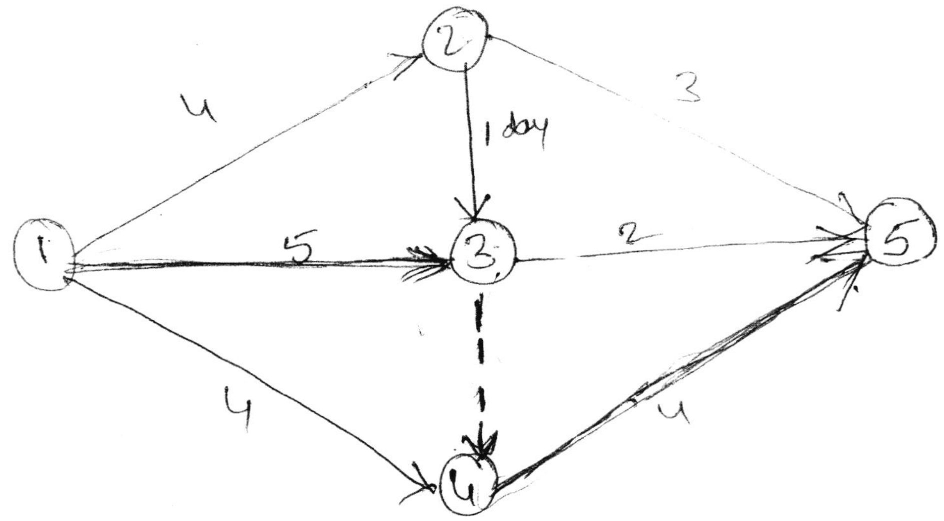
For starting event $T_{E_1} = 0$, At event $T_{E_2} = 0+7 = 7$

$T_{E_3} = 0+3 = 3$ At event 4 $T_{E4} =$ we have more than one path.

1-2-4 = 11
1-4 = 5
1-3-4 = 3

1-2-5 $T_E = 15$
1-2-4-5 $T_E = $ 18
1-3-5 = 9
1-3-4-5 = 10

$T_E = 4$

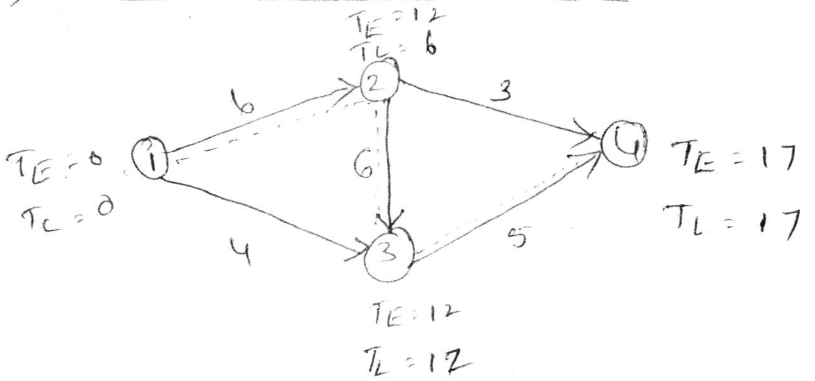


1 - 2 = T_E = 4
1-3-2 = T_E = 6
1-2-3 = 5
1-3 = 5
1-4 = 4
1-3-4 = 5

1-2-5 = 7 ✓
1-3-5 = 7
1-4-5 = 8
1-2-3-5 = 7 ✓
~~1-4-3-5 = 6~~
~~1-3-2-5 = 9~~
1-3-4-5 = 9

(19.1) Example:

I. Crashing 2-3 by 1 day

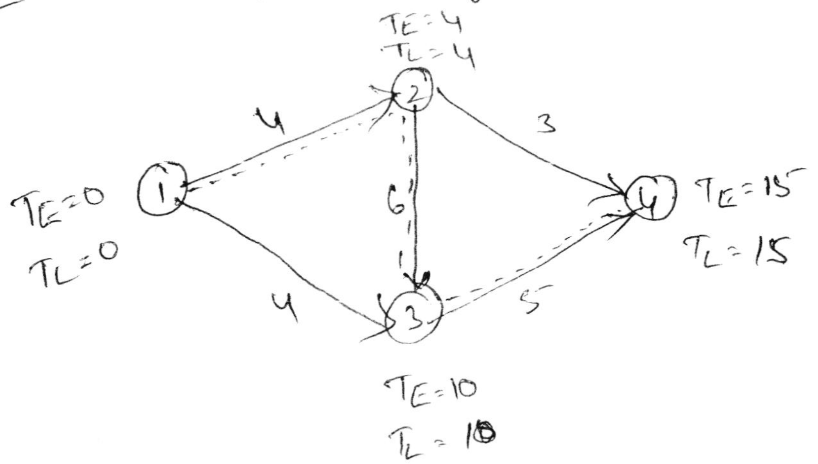


① No change in the Critical path

② The Normal Cost now is 27000 + (1 day × 300)

= 27,300

II. Crashing 1-2 by 2 days



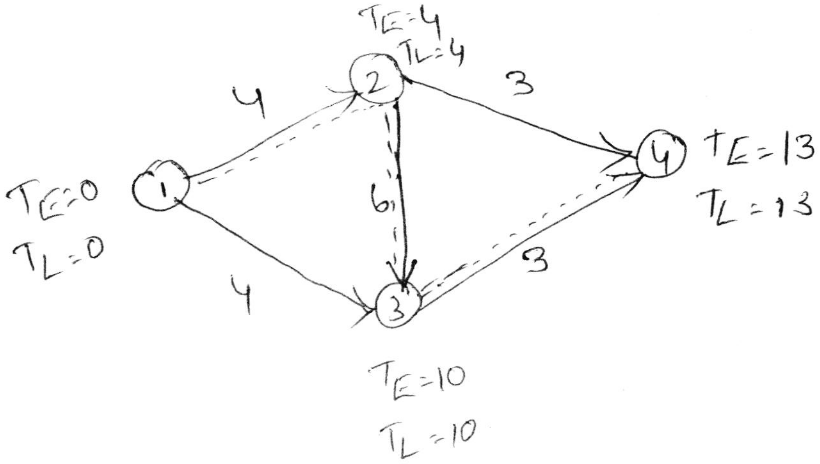
① No change in the C.P.

② Normal Cost

= 27300 + (2 × 600)

= 28,500

III. Crashing 3-4 by 2 days



① No chang in the C.P

② Normal Cost

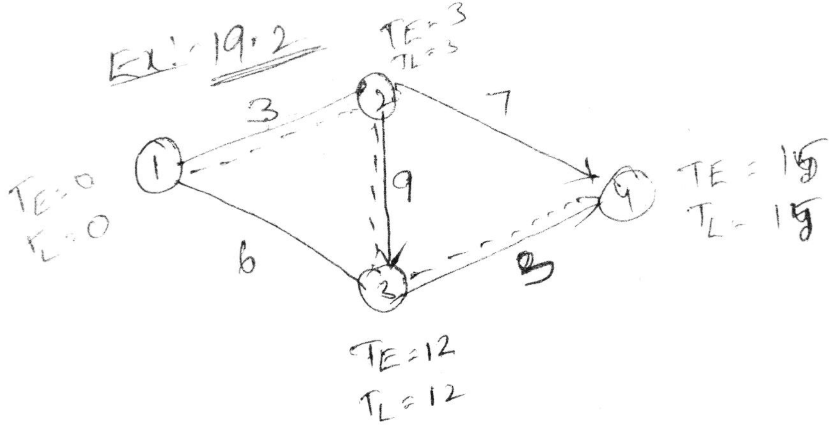
= 28500 + (2 × 750)

= 28500 + 1500

= 30000

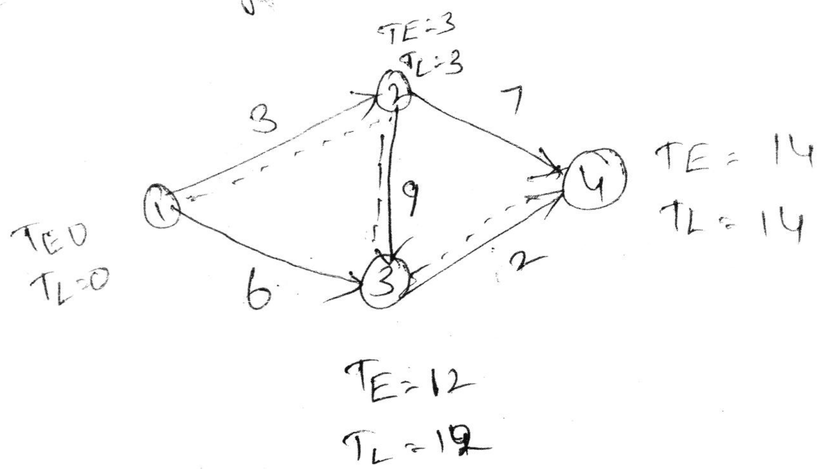
project duration	Normal cost
18 days	27,000
17 days	27,300
15 days	28,500
13 days	30,000

Ex: 19.2



Original Network

I crashing the 3-4 by 1 day



No change in C.P.

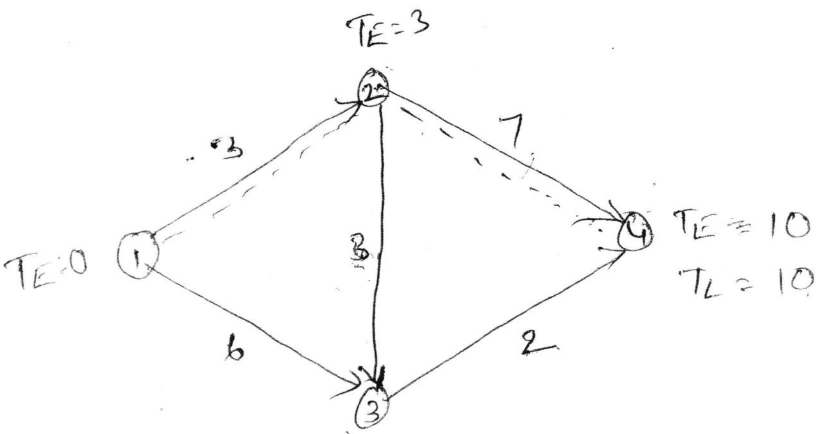
Cost = 33,000 + crash cost 400 + (14 × 600 per day)

I = 41,800

II 49,800

III 56,200

II crashing the 2-3 by 6 days



TE=6, TL=8 — 2 days slack

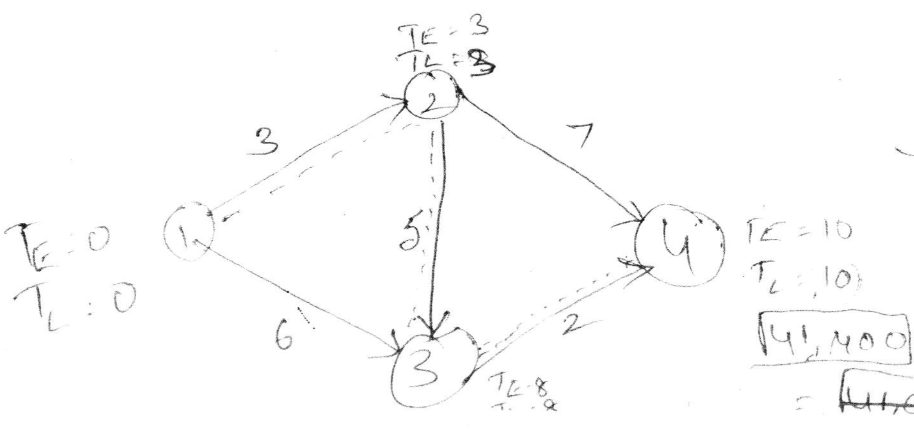
1-2-3-4 TE = 8

1-2-4 = TE = 10 High value

2-4 is not in critical path.

To maintain the original critical path crash 2-3 activity by 4 days only not by 6 days

that means now 2-3 activity takes 5 days.



Cost = 33,400

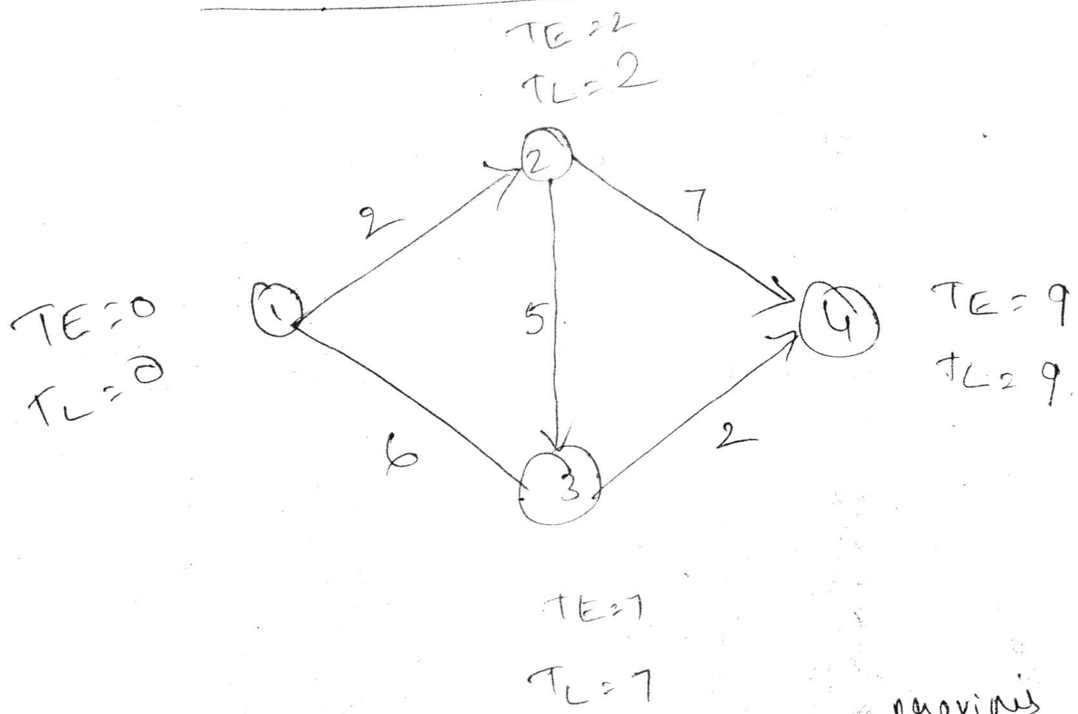
Crash cost = 4 days × 500 = 2000

Indirect overhead = 10 days × 600 = 6000

41,400

= ~~41,000~~

Stage III Crashing 1 – 2 by 1 day



previous crash costs
(3-4) (2-3)
crash cost

Cost of crashing = 33,400 + 2000 + (1 × 1000) + (9 × 600) ← additional exp

= 35,400 + 5400 + 1000

= 41,800.